The Effects of Buyer Information on Consumer and Total Welfare in Competitive Markets

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Abstract

I consider the effect of buyer information on consumer and total welfare in competitive equilibrium with unknown quality types. First, I show that total welfare can decrease when consumers receive a common signal that increases their information about the quality of subgroups of goods. This decrease can happen as a result of two possible channels: decreasing volume of trade and altering the composition of traded goods. Second, I show that the presence of a subset of fully informed consumers can decrease both consumer and total welfare. Next, I present a comparison with the effects of increased information in truth-telling mechanisms, showing how welfare must be non-decreasing in information. Finally, I show that in a market with more sellers than buyers, it is possible for consumers to receive greater surplus from knowing only the underlying quality distribution, instead of having partial, or even perfect, information.

1 Introduction

Many government units, ratings agencies, and independent firms exist to provide buyers with information about the quality of goods for sale in the market. The Bureau of Consumer Protection, the Better Business Bureau, Kelley Blue Book and CARFAX™are all examples of organizations who provide information about business or product quality to consumers. Typically, these entities provide information to either protect consumers, to prevent the unraveling that can be the result of adverse selection, or both. However, it is not clear that all increases in the information available to buyers will serve to increase either efficiency or consumer welfare. In this paper, I consider the effects of increasing the information to buyers on both total welfare and consumer surplus.

I evaluate the impact of several different information increases to buyers. Section 3 examines the case of a common signal to all buyers, which divides the distribution of goods into subgroups. There are two cases: divisions of the original distribution (or of existing subgroups) and shifts of existing subgroups. In both cases, total welfare decreases through two channels: through a decrease in the volume of trade and through a change in the composition of traded goods. Section 4 looks at the consequences of having ”experts”, a subgroup of consumers with perfect information. Section 5 considers what happens to efficiency and consumer welfare when the allocation is based on a truth-telling mechanism instead of a competitive equilibrium. Section 6 examines the issue of consumer surplus in the the previous contexts, showing that consumers may be better off under the lemons market than under perfect information. Section 7 concludes.

This paper provides several key additions to the existing literature. First, it demonstrates how total welfare may not be monotonic in either information or in volume of trade. By altering the composition of trade, informational changes that increase trade may decrease total welfare. Second, the paper expands on the existing discussion of partial buyer information, both by considering a more general signal structure than previous work and by precisely defining conditions for efficiency loss from decreased trade in two quality type markets. Third, this paper allows for a comparison of the effects of a market versus a truth-telling mechanism, with regards to what can be achieved when information increases for the buyers (or mechanism designer). Finally, it addresses specifically the question of consumer surplus, an element of great policy interest that is frequently not discussed in similar work on adverse selection.

1.1 Motivation

Sections 3-4 analyze two different informational environments for the buyers, compared to the standard lemons case:a common signal to all buyers and a fraction of informed buyers. To understand how information changes, consider the classic example from Akerlof (1970) of used car markets. In the standard lemons market, consumers know only the underlying distribution of the quality of cars; they have no way of telling if any particular car is of greater or lower quality than another.

The common signal used in Section 3 is a piece of information that allows consumers to divide goods into subgroups, with new information about the distribution of quality types in the subgroups. In the cars example, this could be some obvious identifying feature such as brand name, location of manufacture, or condition of the paint that gives consumers additional information about the likely quality of a car. If a buyer knows that Brand X has fewer lemons typically than Brand Y, she gains valuable information about expected quality when she identifies the brand of a car. Similarly, if a buyer believes that the outside condition of a car is correlated with the care the previous owner took for engine maintenance, she will believe that the expected quality of an unblemished car is greater than the expected quality of a car with missing paint or scratches.

Section 4 considers the case of a fraction of perfectly informed buyers: some buyers can perfectly determine the quality of any used car, but others are still as uninformed as buyers in the lemons case. In a used car market, the informed buyers might be car experts:

mechanics, used car salesmen, or car enthusiasts. The rest of the buyers are those with no special knowledge to determine car quality.

In each of the following sections, I determine how the additional information of consumers (whether all consumers or just a subset) relative to the standard lemons model affects total welfare, and then Section 6 considers consumer surplus under those information structures. Section 6 shows that in all three situations where buyers have increased information from the standard lemons model (a common signal [Section 3], a fraction of informed consumers [Section 4], and even perfect information), consumers can be harmed by the increase in information.

1.2 Literature Review

Several papers explore similar scenarios to the ones described. Creane (1998) explores a situation similar to the perfect information case of Section 6: a market with unknown quality for buyers where buyers can benefit from a lack of knowledge. However, the nature of the analysis and results are noticeably different here because of the assumption of homogeneous consumers. In Creane, price is determined by the marginal consumer: the price is the value of the indifferent consumer. In Section 6 below, the price is driven by the over-abundance of sellers in the market, and is equal to the marginal cost of the high quality sellers.

Section 4 considers the situation where some buyers have perfect information, and other buyers only know the average quality in the market. Creane (2008) studies a similar question about the effect of buyer information, however his model uses the case of a monopolist with a probabilistic quality. The results of Section 4, however are a result of informed consumers being capable of crowding out uninformed consumers for high quality goods, leaving uninformed consumers with lower expected quality. Under the Creane model, results are not motivated by such crowding out because there is one producer with only one type after quality has been realized.

Several papers focus on the effect of changing seller information, most notably Kessler (2001) and Levin (2001). Kessler (2001) considers the opposite scenario to Section 4: the case where seller information is varying for a subgroup. She finds an uncertain effect from increasing the seller's information: trade (and efficiency) can be increasing or decreasing is seller information. Levin (2001) allows for a more general information structure for the seller: the seller may have a very wide set of possible beliefs, where informational increases are measured by stochastic dominance. He also finds that trade and efficiency are uncertain in increases in seller information.

In addition to considering the effects of increasing seller information, Levin (2001) also considers the effect of additional buyer information on trade in adverse selection markets. Specifically, Section 4 of Levin considers the case of buyers receiving additional public information about sellers that allows buyers to order subgroups of the sellers. There are two key model differences between the analysis in Section 3 here and in the treatment of buyers in Levin (2001). First is the institution under which trade occurs. I consider a competitive market with market clearing, whereas Levin analyzes the case of bilateral trade with a posted market price. This distinction matters because the relative size of buyers and sellers in the market is a requirement for Proposition 2 and the consumer surplus results. The second difference is the structure of the information received by the buyers. In Levin (2001), buyers see whether each good is above or below a certain threshold. Here, however, buyers can receive any signal that subdivides the original group of goods, so long as the resulting subgroups are a partition of the original lemons distribution. The signal structure allows for the same type of information gain as in Levin (2001)[1], but is more general as it allows for some higher quality goods to be in the low signal group and/or some lower quality goods to be in the high signal group.

These differences lead to both different results and different interpretations of the situations described by the models. Under the model of Levin (2001), likelihood of trade (or equivalently expected volume of trade) is perfectly correlated with efficiency. If more(fewer) trades can occur as a result of the information, then the information increases(decreases) total welfare. However, the structure of the signal and the nature of the competitive equilibria in the present paper do not produce the same equivalence between volume of trade and total welfare. I show how an increase in information can increase the total volume of trade, while changing the composition of traded goods in such a way that total welfare decreases.[2]

Because the signals have such different structures, there are different natural interpretations of what signals are being modeled. The bilateral trade problem of Levin (2001) with a cut-off signal is most appropriate for cases where there are many different quality types and buyers can only distinguish between different tiers of quality, such as perhaps the housing market. The signal structure used here works better with fewer quality types, where consumers can find information that refines their beliefs without allowing them to place each good into a specific tier. The original car example given before demonstrates this: Brands X and Y can both produce peaches and lemons, but one brand produces a higher percentage of peaches, which is known to consumers.

Numerous papers have looked at the impact of adverse selection on welfare and efficiency, including the seminal Akerlof paper (1970). One such paper, Rustichini, et al. (1994), considers efficiency with incomplete information. However, the results of the paper depend strongly on an assumption of private values that is violated in the standard lemons model.

[1] With only two quality types, there is no possible additional intermediate information available to buyers in Levin model, any additional information provides the extreme of perfect information. In Section 3.2 I show how with only two quality types the signal structure used in this paper can not only lead to a decrease in total welfare, but the conditions under which this can occur can be completely characterized.

[2] These results, found in Propositions 2 and 3, do depend on any perverse assumptions about the difference in seller cost and buyer value. These results can occur even if each trade is individually welfare-improving.

Similar papers include Hagerty and Rogerson (1985), which looks at a bilateral trade problem with adverse selection, and Gul and Postlewaite (1992), which considers efficiency with asymmetric information asymptotically. The model presented here differs substantially in that it begins with the same starting point as the market described in Akerlof (1970), which can be described neither as a bilateral trade problem nor as a large exchange environment as in Gul and Postlewaite (1992).

Muthoo and Mutuswami (2011) address a related question of the degree of competition on efficiency in lemons market. They consider a situation more similar to my own than other papers, although they do not specify a mechanism for trade. Not specifying a mechanism for trade is one of several ways that equilibria in markets with adverse selection concerns are determined. As summarized by Hellwig (1987), many of the papers analyzing markets with adverse selection consider a Bertrand model of seller decision-making, as opposed to looking at a competitive equilibria. The model presented here is substantially different in that it considers outcomes of competitive equilibria and does not allow for opportunities of signaling or screening behavior.

Other papers have addressed the specific issue of information changes on welfare efficiency in lemons markets. One major branch of literature considers a dynamic market with adverse selection, where information changes as a function of time [for instance, Moreno and Wooders (2013); Bilancini and Bonicinelli (2014)]. However, the welfare changes in this model are all the result of a one-time informational change for the buyers, and are not the result of any dynamic process.

2 Notation and Assumptions

This section provides the basic notation and assumptions used throughout the paper.

2.1 Notation

- ω denotes the index of the quality type of a good. There are a discrete, finite number of quality types, indexed by the natural numbers from 1 to $\overline{\omega}$.
- m is the measure of potential buyers in the market, who demand at most one unit of the sellers' goods.
- n_ω is the fraction of ω quality sellers, who each can sell at most one unit, where $\sum_{\omega=1}^{\overline{\omega}} n_\omega$ is total measure of sellers.
- v_ω is the consumers' valuation of an ω quality good.

- c_ω is the ω quality sellers' value of the item (or cost of selling).
- Ω is the full distribution of goods, the vector of the amounts of each quality type: $(n_1, n_2, ..., n_{\overline{\omega}})$.

2.2 General Assumptions

The following assumptions about buyer valuations, seller costs, and measures of buyers and sellers will be used throughout the paper, unless otherwise noted in a given subsection:

- Buyers are identical: all buyers have the same values for each quality type.
- All sellers of a given quality type ω have the same cost of production.
- All buyers and sellers are risk-neutral.
- v_ω and c_ω are both increasing in ω.
- $v_\omega \geq c_\omega$, $\forall \omega$.

3 Common Signal

In this section, all buyers receive a common signal that divides the distribution of all goods into further subgroups. I consider two types of information changes with the potential to effect trade. First, I examine the case where additional information leads to finer partitioning of the distribution of goods. Second, I analyze the case where the number of subgroups remain the same, but information shifts the mean valuations of the subgroups further apart. Both cases lead to two primary results about total welfare. First, either change in information can lead to falling total welfare through a decrease in the volume of trade, as in Levin (2001).[3] Second, in both cases an increase in trade can lead to a decrease in total welfare, as the composition of traded goods changes as a result of a change in buyer information.

All buyers receive a common signal that they use to update their beliefs about the qualities of goods. Any signal creates a partition of the original distribution Ω. Let S refer to a signal that separates Ω into k different groups: $S_1, S_2, ..., S_k$. The only restriction of the common signal, that the signal must create a partition, requires that: $S_1 \cup S_2 \cup ... \cup S_k = \Omega$. Each signal S specifies a particular distribution of quality for each of its k subsets. For any starting Ω, there will be many different possible signals for a given k.

What types of common signals could occur? Suppose $\Omega = \{2, 2, 2\}$. One possible signal $\hat{S}$ could result in $\hat{S}_1 = \{2, 2, 1\}, \hat{S}_2 = \{0, 0, 1\}$; all buyers can determine the quality of one of the highest quality goods, but all other goods are indistinguishable. Another possible signal

[3] At the end of the section, this is fully characterized for the two quality types case.

S' could be $S_1' = \{1, 0, 1\}, S_2'\{1, 0, 1\}, S_3' = \{0, 2, 0\}$. These signals convey very different information to the buyers, but fulfill the requirement of the common signal: $S_1 \cup S_2 \cup ... \cup S_k = \Omega$.

For buyers, the prior beliefs are those of the standard lemons market: the buyers know the total distribution of quality, the proportions that each quality type is of the entire market. After the signal, the posterior of the buyers provides information about each subgroup analogous to their previous information about the whole distribution. For each subgroup, the buyers are aware of the proportions of all quality types in the subgroup, but do not have information about specific goods in the subgroup.

Since the structure of the common signal is so general, it allows for modeling a variety of real world markets. Examples of such signals in real markets are numerous. Brand names, manufacturing locations, and manufacturing techniques are all ways that consumers may be able to differentiate products based on the likelihood of high or low quality.[4]

The next section considers what can happen when the groups become more finely partitioned (either starting from the original lemons market with all goods in one group or starting from an existing partition of the full market). The next section also shows that the results about total welfare when groups are more finely partitioned, namely that welfare can decrease both because of a decrease in trade and because a change in the composition of trade, can also occur with a shift in the distribution of a signal that does not alter the number of final subgroups in the partition.[5] These decreases in total welfare can occur even if the change in the signal results in subgroups with a greater difference in mean and less variability.

3.1 General Results

I start by giving the general results for an arbitrary number of quality types. In this model of consumer information, there are two ways that informational changes can hurt total welfare: through decreasing the volume of trade and through decreasing the gains to trade of the goods traded (despite an overall increase in volume of trade). I examine these in two contexts: signal refinement, where the signal creates partitions of one or more existing groups, and signal shifts, where an existing signal and resulting distribution is altered to form a new distribution with the same number of subgroups as before. Signal refinements are clear cases of information increases: the existing groups become more refined, and the limit is perfect information, if the goods are not infinitely divisible. Signal shifts can contain several features that are associated with increasing information: increased difference between the means of

[4]Note that this is still different from vertical differentiation. In vertical differentiation, every product from one brand would be preferred by consumers to every product from another brand. Here, the idea is that both brands produce multiple quality types (such as "peaches" and "lemons" in the classic Akerlof example), but one brand has a greater proportion of high quality goods than the other.

[5]This means that the results can hold when moving from signal $\hat{S}$ to signal S', even if both partitions created by $\hat{S}$ and S' produce the same number of subgroups of the full market.

the signal groups and decreased variability inside the signal groups.

3.1.1 Welfare Decreases from Signal Refinements

A signal refinement divides one or more existing groups into further partitions. The introduction of a common signal into the standard lemons market would constitute one such signal refinement: the consumers start with only information about all goods in one group, and after a signal, all goods are partitioned into two or more subgroups. Alternatively, a refinement can occur after consumers have already received one signal (and thus have knowledge about the distribution of two or more subgroups) which causes one or more of the existing subgroups to be further partitioned. When does total welfare decrease, and in what manner?

As previously mentioned, there are two ways that signal refinements can cause a decrease in total welfare. First, the refinement may cause a decrease in the volume of trade (as in Levin (2001)). Second, the refinement may cause a shift in the composition of traded goods; that is, the volume of trade may stay the same or even increase, but the goods traded after the signal is refined may have lower gains to trade, resulting in a reduction of total welfare.

For any signal structure allowed by the model, any subgroup works as its own lemons market. A standard lemons market may have more than one equilibrium: all the comparisons in the following sections will make use of the lemons market equilibrium that trades the most goods.[6] Let the highest traded type for a given lemons market equilibrium (whether the full market or any individual subgroup) be ω^*. This means that trade will occur for all quality types less than or equal to ω^*, and the average value of all goods of quality less than or equal to ω^* is greater than c_{ω^*}.[7] Let the average traded value for a group j, $\frac{\sum_{i \in j}^{\omega^*}(n_i * v_i)}{\sum_{i \in j}^{\omega^*} n_i}$, be denoted u_j.

Finally, consider a signal refinement that divides a starting distribution into two subgroups. Let A refer to the subgroup which traded with higher average welfare gains before the signal refinement, and let B refer to the subgroup with lower average welfare gains before the additional signal.

I consider three scenarios under which signal refinements can decrease total welfare, in ascending order of the minimum number of subgroups necessary for total welfare to decrease. The first case, with volume of trade decreasing, can occur with only a single refinement of the full market into two subgroups. The second case, with volume of trade non-decreasing

[6]Since the primary results show the potential for buyer information to damage total welfare and consumer surplus, the possibility of multiple lemons market equilibria is not problematic. Also, because the signal creates additional lemons markets, the comparison of the markets before and after information change will be a comparison of the highest quantity equilibrium both before and after the signal refinement.

[7]ω^* is the highest quality at which the average value for the buyer is greater than the cost of all sellers with $\omega \leq \omega^*$, since the focus is on the lemons market equilibrium with the greatest quantity traded.

and fewer buyers than sellers, can occur with a single refinement of one of two subgroups, for a total of as few as three subgroups after the refinement. The third case, with volume of trade non-decreasing and no restriction on the relative measures of buyers and sellers, requires a refinement of at least two existing subgroups, for a total of at least four subgroups after the refinement.

First, as in Levin (2001), total welfare may decrease because of a decrease in the volume of trade. Since all trades are assumed to be welfare-improving, any signal that leads to a group of goods traded that is only a partial subset of the previously traded goods must be detrimental to welfare.[8]

A necessary condition for welfare to decrease as a result of a decrease in the volume of trade is:

Condition 1. *$c_{\omega_B^{max}} > u_B$, and there is no $\omega' > \omega_B^{max}$ such that $c_{\omega'_B} \leq u_{B_{\omega'}}$*

Here, $c_{\omega_B^{max}}$ is the cost of the highest quality good traded in subgroup B before the refinement divides the goods into groups A and B. This cost must be greater than the average traded value of the goods in B before the refinement. Additionally, it cannot be possible for trade to occur at a greater quality level than before, which the second half of Condition 1 rules out: there is no quality level greater than ω_B^{max} where buyer value is greater than or equal to seller cost.

Two final groups of goods need to be defined: the goods that are either additionally sold in a subgroup after a signal refinement or that are no longer sold as a result of the signal refinement. If a subgroup trades fewer goods after a signal refinement, let this set of goods be denoted s_j, where j is the subgroup. If a subgroup trades more goods after a signal refinement, let this set of goods be denoted t_j, where j is the subgroup.

Using Condition 1, Proposition 1 gives the requirements for welfare to decrease as a result of a reduction in the volume of trade:

Proposition 1. *Given Condition 1, total welfare will decrease as a result of a decrease in the volume of trade following a signal refinement if $\sum\limits_{i \in s_B} (v_j - c_j) > \sum\limits_{i \in t_A} (v_j - c_j)$*

Proof. Condition 1 ensures that fewer trades must occur in group B after the signal refinement. The requirement of the proposition then is that welfare lost from fewer trades in group B must be greater than any welfare gained from any additional trades in group A.

[8] The following discussion centers on cases where only a partial subset of previously traded goods still trade under the signal refinement. It is possible that the new group of traded goods is smaller than the previous group and gives lower total welfare, without being a subset of the original set of traded goods.

If this is the case, total welfare is decreasing as a result of trade decreasing from the signal refinement.[9] □

Proposition 1 mirrors Proposition 2 of Levin (2001): additional information decreases trade if previously traded goods are lumped into new subgroups such that full trade is no longer possible. If these welfare losses are not offset by greater gains in other subgroups, additional information will cause total welfare to decrease. Unlike in Levin, however, downward-sloping demand is not sufficient to prevent trade decreasing. The first half of Condition 1 is similar to Levin's requirement of upward-sloping demand, but because the signal structure here is not a cut-off value the condition of $c_{\omega_B^{max}} > u_B$ can occur even if demand is downward-sloping. Also, the more general signal structure allowed in this model means that a decrease in the volume of trade can occur with as few as two quality types. Section 3.2 gives a complete characterization of the two quality type case.

For a simple example with three quality types, consider $\Omega = \{4, 3, 2\}$ where buyer values are $v_1 = 1$, $v_2 = 4$, and $v_3 = 8$, and seller costs are $c_1 = 0$, $c_2 = 2$, and $c_3 = 3$. Average consumer value is $3.56 = \frac{4*1+3*4+2*8}{9}$, which is sufficient to ensure all sellers are willing to sell (highest cost is 3). If the measure of buyers is greater than or equal to the measure of sellers, all the goods will trade. Suppose consumers see signal S, which creates the following partition: $S_1 = \{4, 1, 0\}, S_2 = \{0, 2, 2\}$. Average value in group S_1 is $1.6 = \frac{4*1+1*4}{5}$, which is lower than the cost of the type 2 seller. As a result, there is no equilibrium possible that will trade the one unit of quality type 2 from group S_1. The introduction of the common signal has caused total welfare to decrease because of less trade.

Next, I consider the cases of signal refinements changing the composition of traded goods. There are two principal ways that this can occur: the refinement a single subgroup when the total measure of sellers exceeds that of buyers, or the refinement of multiple subgroups, regardless of the relative measures.

Several conditions are necessary for total welfare to decrease as a result of changing trade composition under the signal refinement of a single subgroup. First, the measure of buyers must be less than the measure of sellers:

Condition 2. $m < \sum_{\omega=1}^{\overline{\omega}} n_\omega$

Second, the signal refinement must cause a shift in consumer purchasing as a result of a change in consumer surplus. Let $CS_{B_{max}}$ be the maximum possible consumer surplus when all goods in subgroup B trade.

[9] There is no need for a condition about the relative changes in volume of trade in A and B as a result of the signal. Because A has higher traded welfare under the original equilibrium, added trades must be greater in welfare that the goods traded in A or B. Therefore, for welfare to decrease, the number of trades lost in B must be greater than the number of trades gained in A.

Condition 3. *$CS_{B_{max}} > CS_{A_{max}}$, $\forall CS_{A_{max}}$ where quantity sold in A is between $m - n_B$ and n_A.*

The most consumer surplus that sellers in A can give to buyers without losing any sales is $CS_{A_{max}}$, the difference between the buyer's value and the highest seller's cost in A. The condition requires that sellers in B be able to offer greater consumer surplus to buyers than any subset of sellers in A with quantity greater than $m - n_B$. This means that the sellers in A will not be able to offer prices such that any sellers in B cannot trade. This condition ensures that buyers wish to switch from purchasing all units from A and some units from B to purchasing all units from B and some units from A.

Proposition 2. *Given Conditions 2 and 3, a signal refinement of group D will result in a decrease in total welfare with the volume of trade non-decreasing if:*

1. $\sum_{i \in s_A} (v_i - c_i) > \sum_{i \in t_{BA}} (v_i - c_i) + \sum_{i \in t_{BB}} (v_i - c_i)$ *and*

2. $n_{s_A} \leq n_{t_{BA}} + n_{t_{BB}}$

Proof. Conditions 2 and 3 together ensure that consumers will switch from purchasing from group A to group B. Condition 2 requires that there always be unsold goods, since the measure of buyers is less than the measure of sellers. Condition 3 means that the unsold goods move from group B to group A.

The two requirements in the proposition guarantee that welfare will be decreasing and trade will be non-decreasing. The first requirement is that gains to trade lost in group A as a result of consumer switching must be greater than gains to trade gained in B from consumer switching. The second requirement is that the number of trades gained in group B is greater than or equal to the number of trades lost in group A.

In order for total welfare to decrease without decreasing the volume of trade, trade must shift from the group where higher values were traded before the signal, A, to the group which had lower traded values, B. This is the reason behind the direction of the inequalities in the requirements: group A must be losing trades (and thus welfare) and group B must gain at least as many trades as group A loses. Taken together, these requirements mandate that total welfare must be decreasing while trade is non-decreasing. □

Proposition 2 yields the surprising result that volume of trade is not a reliable indicator of total welfare, as it was in previous models such as Levin (2001). Refining the signal for even one subgroup can lead to a decrease in the number of goods with higher gains to trade sold, while the overall level of trade increases. To see an example how a single signal refinement can cause total welfare to drop through a change in the composition of trade, see example A.1 in the Appendix.

If there are already two subgroups before the signal refinement, Proposition 2 can occur if group A receives a signal refinement which leads to the conditions above. This can occur with as few as three quality types, and no additional restrictions on the gains to trade. Moreover, this result can also occur with a single refinement when starting with the full distribution, if there are more than three quality types and gains to trade are not monotonically increasing in quality.

Finally, signal refinements can lead to a decrease in total welfare through a change in the composition of trade more generally when multiple subgroups receive further refinement. Although this is a greater restriction on the structure of information before and after the signal than in the previous case, no restriction is needed for the relative measure of buyers and sellers. Therefore, though the information structures are more limited, the market structures in which this can occur are more general. One primary condition is necessary:

For these cases, let group AA be the subgroup after additional signal refinement of group A which had higher average traded value before the refinement, AB be the subgroup after additional signal refinement which had lower traded value before the refinement, and likewise BA and BB for group B.

Condition 4. *$c_{\omega_{AB}^{max}} > u_{AB}$, and there is no $\omega' > \omega_{AB}^{max}$ such that $c_{\omega'_{AB}} \leq u_{AB_{\omega'}}$*

$c_{\omega_{AB}^{max}}$ is the cost of the highest quality good traded in subgroup AB before the refinement further divides group A into AB and AA. This cost must be greater than the average traded value of the goods in AB before the additional refinement. Additionally, it cannot be possible for trade to occur at a greater quality level than before, which the second half of Proposition 2 rules out: there is no trade at a quality level greater than ω_{AB}^{max} agreeable to both buyers and sellers.

Using Condition 4, it is possible to define when the refinement of multiple subgroups will lead to a decrease in total welfare, with the volume of trade non-decreasing:

Proposition 3. *Given Condition 4, signal refinements of groups A and B will result in a decrease in total welfare with the volume of trade non-decreasing if:*

1. $\sum_{i \in s_{BB}} (v_i - c_i) + \sum_{i \in s_{AB}} (v_i - c_i) > \sum_{i \in t_{BA}} (v_i - c_i) + \sum_{i \in t_{AA}} (v_i - c_i)$ *and*
2. $n_{s_{BB}} + n_{s_{AB}} \leq n_{t_{BA}} + n_{t_{AA}}$

Proof. Under Condition 4, some trades in subgroup AB that were possible before the signal refinement are no longer possible. The first requirement of the proposition ensures that the lost welfare from these transactions (and any lost transactions in BB) must be greater than the added trades in groups AA and BA. The second requirement ensures the number of added trades in AA and BA are greater than or equal to the number of lost trades in BB and AB. Together, these requirements guarantee that total welfare must be decreasing and the volume of trade is non-decreasing. □

When multiple subgroups are further refined, total welfare can decrease even when the total volume of trade is non-decreasing. This happens through a decrease in higher welfare trades in group A and an increase in lower welfare trades in group B. An example of this is provided in A.2.

3.1.2 Welfare Decreases from Signal Shifts

The previous section considered the way in which refining, or further partitioning, consumers' information of quality could decrease total welfare. This section considers the broader case of changes to existing common signals. The particular information change considered here is an increase in the difference of the mean values of the subgroups, as a result of a change in the signal. For instance, consider a government rating agency that publishes a single grade for businesses to inform consumers about some aspect of business quality. The agency has the option to alter the criteria by which they assign grades. The agency may desire to issue new grades if the new criterion provides greater mean differences between the graded groups.[10]

Suppose buyers receive a signal that puts all goods into one of two groups: higher than the lemons average value (C) and lower than the lemons average value (D). Let C' be the group with higher average value after the signal shift, and D' the group with lower average value after the shift. An increasing mean difference implies that the difference between the average values of groups C and D will be moving further apart: $v_{C'} > v_C > v_D > v_{D'}$. The purpose of considering such signal changes is to examine other situations that may be perceived as information increases: namely when mean values of the signal groups move further apart and the variability of the subgroups diminish. With two quality types (as in Section 3.3), either of these two requirements are met when information increases. Regardless of the number of quality types, increasing the difference of the means, especially when decreasing the variability of the subgroups, moves information away from the starting point of the lemons model with one distribution of quality types. This section shows total welfare can decrease under these conditions, just as it did under signal refinement.

An example can be used to show how this might happen. Suppose that once again there are three quality types, and that group C has all of the high quality, some of the medium quality, and some low quality, and group D has the rest under the first signal. Suppose that values and costs are such that under the first signal, both signal groups are able to sell both of the lower types, but not the high type (for C). Now consider a new signal

[10]The information change considered in this section would assume that buyers cannot see or remember grades from the old criterion.

with the same general distribution as the old signal, only moving several low quality goods from C to D. Note that this satisfies the condition of an increase in the difference of the mean values: moving several of the lowest quality from C to D guarantees the $v_{C'} > v_C$ and $v_D > v_{D'}$. Under the new signal, the average value of D' may be so low that the medium quality sellers may be unable to trade. Though $v_{C'} > v_C$, it may not have been a sufficient increase to bring high quality sellers into the market. In this case, the volume of trade decreases from the old signal to the new, because group D' is unable to trade medium quality, and no new trades have been added. Example A.4 gives a numerical example of this case.

Once again, this trade decrease can happen because of either group C or D. Take the same scenario as the previous paragraph, and suppose instead that the low signal group had one high quality good under the old signal. Under the new signal, the one medium quality good and the only high quality good that were in group D get moved to group C'. Once again, this means $v_D > v_{D'}$ and $v_{C'} > v_C$. However, if the additions were not sufficient first to bring the high quality into the market and, once the high quality has left the market, also not sufficient to bring in the medium quality of group C', then trade has decreased. The medium quality good that switched groups has gone from being traded under the old signal to not traded under the new signal, with no new trades added to offset the loss.

The other way that welfare can decrease after an increase in the difference of the mean values of the subgroups is through a change in the composition of traded goods. How does this occur?

It is easy to understand how this might occur when the difference between the mean values of the subgroups decreases. For instance, if there were three quality types, if group C were to lose some medium quality to group D after a new signal, then it is certainly possible group C' would no longer have sufficient average value to sell high types while group D' would be able now to sell the medium types it may not have previously been able to. The volume of trade may very well increase, but if the gains to trade are much greater for the high type than the medium, this would result in a loss of total welfare. For a more detailed numerical example, see A.6.

However, this is possible even if the difference between the mean values of the subgroups is increasing.

Proposition 4. *A signal shift from signal S to signal S' that increases the difference between the average values of the subgroups ($v_{C'} > v_C > v_C > v_{C'}$) leading to increased trade may cause total welfare to decrease, if u_A increases and u_C decreases as a result of the signal shift.*

Proof. Suppose instead that u_B decreased. In this case, there will be no additional goods sold that were lower than u_A. If B loses average traded value, it cannot gain additional trades: lower average quality means higher quality types in B cannot sell - adverse selection is getting worse.

Likewise, suppose instead that u_A increased. If A gains average traded value, it will not sell any more goods that are lower than the previous average, rather it can only sell above average quality. This will result in an increase in total welfare.

Therefore, it is not possible for total welfare to decrease with an increase in both the difference of the mean values ($v_{C'} > v_C > v_C > v_{C'}$) and volume of trade unless B increases its average traded value, and A decreases its average traded value.

□

A.4 shows such shift leading to decreased total welfare when $C = B$ and $D = A$, and A.5 where $C = A$ and $D = B$.

How does this occur? There are two possible cases, group C has a higher starting average traded value (is group A), or group D does.[11] Here, the intuition for the simpler case, where C starts out with a greater average traded value, is provided. Since C starts out higher, what is needed for the proposition to be met is for C to lose quality types in the middle to D. If the quality types are low enough, the average value for C will increase and for D will decrease, meeting the condition of the increase in the difference of the mean values. However, it is possible that while the average value in C is increasing, the loss of goods in the middle levels of quality will lead to a more severe adverse selection in group C. For instance, under the first signal, the lowest four quality types might have been trade-able, but after losing many of those medium quality goods to D, only the two lowest quality types are trade-able. In group D, suppose only the lowest type was trade-able under the first signal, but now the addition of new medium quality types make the the two lowest types now trade-able. If D had enough of the second-lowest quality type, the new signal may result in an increase in trade. However, if there are much greater gains to trade in the quality levels that used to sell in C but no longer do, total welfare could decrease. For numerical examples, see A.4 (where the mean difference increases and variability of both subgroups decreases) and A.5 (where the mean difference increases but the variability of one subgroup increases).

What does this mean in practice for those concerned with markets subject to adverse selection? One important theme of this section is that neither informational increases nor increases in the volume of trade are sufficient measures of total welfare in adverse selection markets. When determining the level of success of a given policy or market intervention, it is not sufficient justification to demonstrate that consumers have better information or even that more trades are now occurring.

[11] A.4 provides an example of group D starting with a higher average traded value, and A.5 an example of C starting higher.

3.2 Two Quality Types

With the general results established for an arbitrary number of quality types, I completely characterize the conditions under which the first primary result holds for the two quality types case: when does total welfare fall when information increases? Unlike the other results of this section or the results of Levin (2001), this can occur with as few as two quality types. There are l low quality goods and h high quality goods, with the restriction that $m < l + h$, namely that the measure of buyers is less than the total measure of sellers. Additionally, it is assumed that all trades are welfare non-decreasing (buyer values always greater than seller costs) and that the gains to trade are higher for the high quality good. Finally, I assume that the highest quantity lemons market equilibrium allows m trades: the average value of the m lowest quality goods is higher than the cost of the high quality.

The following assumption is necessary for decreased total welfare from a decrease in trade:

Assumption 1. $n_{D_l} > l + h - m$

This assumption states the measure of high quality goods in D (the group with the lower expected quality than the standard lemons) must be greater than the difference between the total measure of sellers and the measure of buyers. This assumption means that if high quality goods in the low signal group sells, then fewer than m trades must occur.

Let v_{D^*} be the consumer valuation of purchasing from the D group after $l + h - m$ high quality sellers who were in the D group drop out of the market. With l_D low quality goods and $h_D-(l+h-m)$ high quality goods in group D, $v_{D^*} = \frac{l_D}{l_D+h_D-(l+h-m)} v_L + \frac{h_D-(l+h-m)}{l_D+h_D-(l+h-m)} v_H$. This leads to another necessary assumption:

Assumption 2. $v_{D^*} < c_H$

This assumption is necessary for high quality goods in the low signal group not to trade. Under Assumption 2, there are few enough goods in group A that all can trade: if $h_D > l + h - m$, then $l_C + h_C < m$.[12] Since consumers have higher average values for the goods in group C than in group D, all goods in group C will trade, as in perfect information the highest quality types always trade. This means that some sellers in group D will be unable to trade, at least $l + h - m$ of them. Since high and low quality types inside one signal group are indistinguishable, the price for the low signal group will have to be such as to make at least $l + h - m$ sellers unwilling or indifferent to selling. This price will be the cost of the high quality sellers, c_H.

After the first $l + h - m$ high quality sellers in group D drop out of the market, expected value for the group becomes v_{D^*}. Assumption 2 states that this value is below the cost for

[12] Recall that $h_C + l_C + h_D + l_D = l + h$

the high quality sellers. Thus, all remaining high quality sellers in the low signal group drop out of the market.

Once all the high quality sellers in the low signal group drop out of the market, some buyers will be unable to purchase because the measure of high quality sellers in the low signal group is assumed to greater than the difference between sellers and buyers Assumption 1. With some buyers unable to purchase, all buyers will receive consumer surplus of zero. Therefore, the low quality sellers in the low signal group will sell at a price equal to buyer value, v_L. Additionally, the sellers in the high signal group will sell at a price equal to expected value of the high signal good, v_C.

Proposition 5. *If Assumptions 1 and 2 are satisfied, then the equilibrium with highest quantity sold in the standard lemons market yields higher total welfare than when consumers receive an informative common signal.*

Proof. Total welfare depends on the measure of goods sold and the quality of those goods. In the standard lemons market, the goods traded are all l low quality goods and $m - l$ high quality goods, under Assumption 1. In the common signal case, consumers purchase all goods in the high signal group and all low quality goods in the low signal group. However, because none of the high quality goods in the low signal group can trade under Assumption 2, the total measure of goods sold is less than the measure of consumers (Assumption 1), so the measure of goods sold is less than m, the number sold in the standard lemons. Because all low quality goods are sold in both cases but fewer goods are sold in the common signal case, it must be the case that fewer high quality goods are sold than in the lemons market. Since fewer trades are made and the average quality is strictly lower than in the lemons market, the common signal case has strictly lower total welfare than the standard lemons market case. □

Proposition 5 provides the complete characterization of necessary and sufficient conditions for total welfare to decrease as a result of the introduction of a common signal. In the two quality type case, this decrease results only from a decrease in the volume of trade. A.3 provides an example of how this total welfare decrease can occur.

4 Individual Buyer Information

In many markets for products for varying levels of quality, buyers will have varying degrees of experience with or information about a particular product. For instance, in a used car market a mechanic may know much better than an accountant what the quality of a particular used vehicle is. In this section, I consider the case of a fraction of experts, denoted r, who have perfect information about the quality of each car. The remaining $m - r$ buyers are the same as in the standard lemons model: the only information they possess is the underlying distribution of quality. As in the end of the previous section, there are two quality types,

l low quality goods and h high quality goods. It is also still assumed that $m < h + l$: the measure of buyers is less than measure of sellers.

Suppose that $m - r > l$, the measure of uninformed buyers is greater than the measure of low quality goods. If there is such a number of uninformed consumers, only one condition is necessary for total welfare to decrease compared to the standard lemons model:

Condition 5. $\frac{l*v_L+(m-r-l)*v_H}{m-r} < c_H$

Proposition 6. *If $m-r > l$ and Condition 5 is satisfied, then total welfare with a fraction of informed consumers is less than or equal to total welfare under the standard lemons market.*

Proof. Since there are more sellers than buyers and more uninformed buyers than low quality sellers, informed buyers will always be able to purchase from high quality sellers. Informed buyers would always be able to offer ϵ more than uninformed sellers to individual high quality sellers in order to guarantee receiving a higher quality good. Therefore, all r informed sellers receive high quality goods.

Uninformed buyers know that informed buyers receive high quality goods. Therefore, if $m - r$ goods are still being offered, those goods must consist of l low quality goods and $m - r - l$ high quality goods. If expected value with l low quality goods and $m - r - l$ high quality goods is less than c_H (Condition 5), then uninformed consumers will be unwilling to purchase at price c_H, the price that brings high quality sellers into the market. In this case, l uninformed buyers will buy low quality cars at a price of v_L and $m - r - l$ uninformed consumers will be unable to purchase. As a result, all uninformed buyers receive consumer surplus equal to zero. Also, $m - r - l$ fewer high quality goods sell than in standard lemons model. $m < h + l$ guarantees that under lemons market, l trades occur with non-negative consumer surplus. Therefore, in this case the increase in information to a select group of buyers results in lower total welfare than the standard lemons market, as the same number of low quality goods trade but fewer high quality goods trade. □

5 Truth-telling Mechanisms

Much of the economic literature addressing issues of adverse selection and incomplete information move away from competitive equilibria to consider cases of Bertrand pricing (Hellwig, 1987) and truth-telling mechanisms. Since I consider markets with many sellers each with no market power, the Bertrand pricing models are not particularly relevant; however, mechanisms may be as valid as the competitive equilibria. Because of the revelation principle [Gibbard (1973), Green and Laffont (1977), Dasgupta, Hammond, and Maskin (1979), and Myerson (1979)], it suffices to look only at truth-telling direct mechanisms. In the next section, I determine what is achievable by a truth-telling mechanism and compare the efficiency to the competitive equilibria, in both the standard lemons model and the common signal condition of Section 3.

The mechanism designer in both cases is limited to the information of the buyers: only knowing the underlying distribution in the standard lemons model and knowing the distributions of both signal groups in the common signal case. The designer can set the prices and probabilities of trade, with the standard restrictions of the participation and incentive compatibility constraints of the sellers. For the risk-neutral buyers, the mechanism only needs to satisfy their participation constraint: the average value of goods allocated to buyers after the mechanism must be greater than or equal to the average price paid.

Let x be the probability that a good called "low" is traded and let y be the probability that a good called "high" is traded. Clearly, given truth-telling, the greater y is, the greater the efficiency of the final allocation.

The general assumptions on the two quality type case still hold: specifically, there are m buyers and $l + h$ total sellers, where $m < l + h$. This places a restriction on x and y: in a truth-telling equilibria: $x * l + y * h \leq m$, the measure of goods to be traded must be less than or equal to the measure of buyers. To maximize efficiency, this condition should be strict.

As before, p_L and p_H are the price of a low quality and high quality good, respectively.[13] For the mechanisms considered here, the price is only paid to the seller if trade occurs.[14] For the sellers then, the participation constraints require that $p_L \geq c_L$ and $p_H \geq c_H$.

5.1 Truth-telling in the standard lemons model

What can be achieved in the standard lemons model through the use of a mechanism that incentivizes truth-telling?
The incentive compatibility constraint for the low type is:

$$x(p_L - c_L) \geq y(p_H - c_L) \tag{1}$$

and for the high type is:

$$y(p_H - c_H) \geq x(p_L - c_H) \tag{2}$$

Proposition 7. *There is no incentive compatible mechanism that induces truth-telling where $y > x$.*

Proof. Solving eq. (1) for $x * p_L$ gives:

$$x * p_L \geq y(p_H - c_L) + x * c_L$$

[13] The focus is on truth-telling equilibria, therefore no distinction is made between the price of low quality goods and the price of goods reported as low quality. In other equilibria, the distinction would become relevant.

[14] Other models sometimes use a price that is always paid to the seller. The prices used here can be transformed into similar prices by multiplying the prices by the probabilities of trade.

This can be substituted into the second constraint, eq. (2) to give:

$$y(p_H - c_H) \geq y * p_H + x * c_L - y * c_L - x * c_H$$

Subtracting $y * p_H$ from both sides and simplifying yields the following condition:

$$0 \geq x(c_L - c_H) + y(c_H - c_L) \tag{3}$$

Since it was assumed from the start that the cost of the high quality good is greater than the cost of the low, $c_L - c_H$ is negative. Thus, for the entire expression on the righthand side of the inequality to be below zero, x must be greater than y. Therefore, there is no incentive compatible mechanism that induces truth-telling where y is greater than x.[15] □

Given Proposition 7, if y cannot be greater than x, the most efficient mechanism would then have $y = x$, and have l goods sold. Looking at the incentive compatibility constraint for the low quality sellers, this could only occur if the prices were the same, $p_L = p_H$. This will be the second best mechanism, if it also satisfies for the participation constraint of the buyers (or if budget balancedness is not required by the designer). The lowest possible price (both p_H and p_L) that the mechanism designer can charge is c_H; any price lower, and the high quality sellers would be unwilling to participate. This leads to the participation constraint of the buyers, which requires that:

$$\frac{xm}{xl + yh} v_L + \frac{yh}{xl + yh} v_H \geq p_H \tag{4}$$

Since the designer wants to maximize the number of transactions, $xm + yn = i$. Also, because $x = y$, Equation (4) can be rewritten as:

$$\frac{x}{m}(l * v_L + h * v_H) \geq p_H \tag{5}$$

Proposition 8. *Given Equation* (5) *holds and* $p_H \geq c_H$*, the truth-telling mechanism with the greatest efficiency is one where* $x = y$*,* $x(l + h) = m$*, and* $p_H = p_L$*.*

Proof. If Equation (5)is satisfied, the consumers' participation constraint is satisfied. If $p_H \geq c_H$, then $p_L = p_H \geq c_H > c_L$ and the participation constraints of both types of sellers are satisfied. Finally, as shown in Proposition 7, the maximum proportion of high quality goods that can be sold under a truth-telling mechanism must be equal to the proportion of low quality goods, and will maximize efficiency when every buyer receives one good. □

With Proposition 8, the most efficient truth-telling mechanism with the maximum consumer surplus can be defined. This will be the mechanism that gives as little of the gains to trade to the sellers as possible, while maintaining the participation constraints. This requires setting p_H (and thus p_L) equal to c_H.

[15] If x and y are directly affected by the reports of the sellers, it is possible for y to be slightly larger than x. For instance, if the mechanism committed to trading a specific measure of reported high and reported low quality goods trading, it would conceivably be possible to have $y > x$, if after deviation from one of the low quality sellers, the new y' is less than x.

Corollary 1. *Given Proposition 8 holds, the most efficient truth-telling mechanism that maximizes consumer surplus is one where $x = y$, $x(l + h) = m$, and $p_H = p_L = c_H$.*

How does the truth-telling equilibrium defined by the mechanism compare to the competitive equilibria of the standard lemons model? Consider a market where Assumption 1 holds and there is only one competitive equilibrium. This allows for a comparison between one competitive equilibrium and the different truth-telling equilibria (which only vary based on the division of the gains to trade).

First, note that the truth-telling mechanism must be more efficient that the competitive equilibrium of the standard lemons model. At the competitive equilibrium, all l low quality goods are traded, and $m - l$ high quality goods are traded. This means that all the unsold goods ($l + h - m$ of them) are high quality goods. Under the truth-telling mechanism, equal proportions of high and low quality goods are traded (or not traded). Since this means some low quality goods are not traded but every consumer receives one good, the low quality goods are replaced by a higher quantity of high quality goods than in the competitive equilibrium. Since the same number of goods are traded in both equilibrium types but more high quality are traded in the truth-telling mechanism, the equilibrium of the truth-telling mechanism is more efficient in these situations than the competitive equilibrium.

Also, note that consumers receive greater consumer surplus under the mechanism of Corollary 1: consumers pay a price for each good equal to c_H, the same as in the competitive equilibrium, but since the average quality of goods is higher, the total consumer surplus is also higher under the truth-telling mechanism.

Finally, the question remains what the most efficient mechanism would look like if Equation (5) does not hold. Recall that if Equation (5) does not hold, the participation constraint of the buyers will not hold. In this case, the price of the low quality good will have to be dropped below p_H. To prevent deviation from the low quality sellers, the probability of selling as a low quality seller must be higher than the probability of selling as a high quality seller, $x > y$. However, if x stays below 1, the truth-telling mechanism be more efficient that the competitive equilibria.

If, however, the low quality sellers still want to deviate when $x = 1$, $x * l + y * h = m$, and p_L is set as high as possible while satisfying the participation constraint of the buyers, then the truth-telling mechanism will require an even lower y, which means that some buyers will be unable to receive goods. If this occurs, then the truth-telling mechanism actually performs worse that competitive equilibrium with maximum quantity sold: in both cases, all low quality goods sells, but in the competitive equilibrium the remaining $m - l$ buyers receive high quality goods and in the truth-telling mechanism some of the $m - l$ buyers receive nothing.

5.2 Truth-telling in the Common Signal Case

Next, I consider the same situation as Section 3: an informative signal, this time viewed by the mechanism designer, that allows the designer to distinguish between groups with greater and lower expected quality than the average quality of all goods together.

Since the designer can distinguish between the groups, she will not be limited to one mechanism; rather she can design separate mechanisms for the high signal and low signal groups. Each of the two mechanisms have the same limitations as the mechanism described in Proposition 8. The probabilities of trade must be the same, and the prices must also be the same if the designer wishes to incentivize both types to trade. In some cases, the designer may want the high quality sellers in the low signal group not to trade, in order to lower the price paid to low quality sellers in the low signal group.

In trying to maximize efficiency, the designer will wish to ensure that all the goods in the high signal group sell (this was group A in Section 3). If there are any consumers remaining, then the remaining number of necessary goods will be traded from the low signal group (group B). Does the improvement in information allow for a more efficient mechanism?

Under the competitive equilibria of Section 3, it is possible to have lower total welfare with the common signal. However, with the mechanism receiving the signal will always increase efficiency of the truth-telling equilibrium.

Proposition 9. *A truth-telling mechanism designed with more finely partitioned information about seller quality must result in at least as much efficiency as a truth-telling mechanism with less information, and may result in an increase in efficiency.*

Proof. The reasoning behind this proposition is simple: a mechanism designer can always implement the same trading mechanism as with less information, by ignoring additional information. For the two quality case, this is the mechanism outlined in Proposition 3, the second-best mechanism.

To show that it may be possible to increase in efficiency, consider a case where $v_L \geq c_H$, so that trade prices of c_H in the mechanism will permit both the participation constraints of the sellers and buyers to hold. The mechanism designer will set the mechanism so that all possible goods trade: either m goods if the measure of goods in A is greater than or equal to l, or $h_A + l_A$ goods (all the goods in group A) if the measure of goods in A is less than m. If all goods in A are traded with some consumers not receiving goods, the remaining goods are traded from group B, with price c_H and the same probability for both high and low types, where the probability is such to allow for a final total of m traded goods.

In this case, with different mechanisms for the high and low signal types, all m buyers still receive goods, but a higher percentage are taken from group A than group B. Since group A had expected quality greater than the whole distribution without any signal, the quality traded increases under the new mechanisms. Since efficiency increases when consumers are able to receive high quality types instead of low, efficiency is increasing in the additional information of the mechanism designer. □

Why is this different from the competitive equilibrium result? While ignorance can be beneficial for consumers as a group (and for total welfare) under competitive equilibrium, each individual consumer could hurt herself if she ignores additional information. As a result, consumers become willing to pay less for a good in the low signal group, which as shown in Proposition 5 can cause high quality sellers to drop out of the market and lead to an inability to buy for some consumers and a resulting loss in total welfare. However, the mechanism designer does not face this challenge; instead, she can choose to ignore the signal and enforce the same allocation as before the signal, ensuring efficiency is not harmed by additional information.

6 Consumer Surplus Effects

Previous sections have considered only the impact of information on total welfare. However, since providing additional information to buyers is often suggested as a means of benefiting consumers in markets with adverse selection concerns, it is important to understand when additional information will benefit consumers, and when instead it could harm them. This section looks specifically at consumer surplus for the cases already analyzed. First, I look at the two most extreme cases, the lemons market and perfect information, and show that the lemons market can yield higher consumer surplus than perfect information. Then, I consider the intermediate cases of the common signal and a fraction of informed buyers, and show how once again consumers can be better off as a group without additional information compared to the lemons market.

6.1 Lemons Market and Perfect Information

I start with the case of only two quality types. The notation follows Section 3.2, with l low quality types and h high quality types, and the assumption that $m < l + h$: the measure of buyers is less than the measure of sellers. Additionally, it is again assumed that in the standard lemons market, m goods can be traded.

For the existence of these equilibria where the high type is traded, the following condition must be true:

Condition 6. $\frac{l*v_L+(m-l)*v_H}{m} \geq c_H$

$\frac{l*v_L+(m-l)*v_H}{m}$ represents the expected quality of purchasing when m goods are being offered in the market, sufficient goods to meet all potential consumer demand. All low quality goods are offered, and $m-l$ high quality are also offered. If Condition 6 is met, there will be between one and three equilibria, at least one of which will include some high quality goods trading.

If $v_L > c_H$ (implying Condition 6 is strong), there will be only one equilibrium in the standard lemons model. Since the value for a low quality is greater than the cost of producing a high quality good, as the price rises to c_H, all buyers stay in the market and high quality sellers begin to enter the market. The only competitive equilibrium ($Q_s = Q_d$) is when l low quality goods and $m - l$ high quality goods are traded, at a price of c_H. In this equilibrium, consumer surplus per buyer is $\frac{l*v_L+(m-l)*v_H}{m} - c_H$: the expected value of the goods being offered minus the price paid.

If Condition 6 is only weakly satisfied (implying $v_L < c_H$), there are two equilibria in the lemons model: an equilibria at a price of v_L where l low quality goods trade, and an equilibria at a price of c_H where l low quality goods and $m - l$ high quality goods trade. In both these equilibria, consumer surplus is zero: in the first, consumers pay a price of v_L for a good with expected value v_L and in the second they again pay a price equal to value (since Condition 6 is weak).

If $v_L < c_H$ and Condition 6 is strong, there are three equilibria in the lemons model. The first is the same as in the previous case: m low quality goods trade at a price of v_L. The second equilibrium occurs at a price of c_H, when $\frac{l}{l+\lambda}v_L + \frac{\lambda}{l+\lambda}v_H = c_H$, where λ is the measure of high quality goods that makes the equality hold. Since Condition 6 is strong, it must be the case that λ is less than $m - l$, and this equality will exist for a feasible measure of traded goods. The third equilibrium occurs where all buyers receive a good, l low quality goods and $m - l$ high quality goods are traded. In the first two equilibria, consumer surplus is zero, as the value of traded quality is equal to price. In the last equilibrium, there is positive consumer surplus, the size of which is determined by the difference between the left and right sides of the inequality in Condition 6.

In this section, I provide the necessary and sufficient conditions for buyers to benefit from a lack of information, with two quality types.

Several conditions are necessary for buyers to benefit from the inability to distinguish quality types. Several additional assumptions are necessary here: $m > h$ and $m > l$.

If $m < h$, in the perfect information equilibrium only high quality goods sell, at the cost of production, c_H. In this case, the surplus of each buyer is $v_H - c_H$, which is also the maximum gains to trade from a transaction. Since every buyer receive the maximum possible surplus, consumer surplus can never be increased from the perfect information equilibrium.

$m > l$ is necessary because if the measure of buyers is less than the measure of low quality sellers, the only possible lemons market equilibrium is where all buyers buy from low quality sellers at a equilibrium price equal to the cost of producing the low quality good. In this case, each buyer gets surplus equal to $v_L - c_L$.

As mentioned above, under perfect information the consumer surplus of all buyers is $v_L - p_L$ or equivalently, $v_L - c_L$. For consumer surplus to be higher in the lemons market than under perfect information, the average buyer must receive consumer surplus greater than $v_L - c_L$. To determine whether this is possible, first find the lemons market equilibrium that maximizes quantity sold. The previous section detailed all the possible lemons equilibrium given the starting assumptions. In any equilibrium where consumers do not all purchase, the consumer surplus is zero. Therefore the only equilibrium where it is possible for consumer surplus to be greater than the perfect information case is when m consumers purchase.

As shown above, at equilibria where all m buyers can purchase, expected value per buyer will be $\frac{l}{m}v_L + \frac{m-l}{m}v_H$. Thus, consumer surplus will be higher under the lemons market (where consumers have no knowledge of the quality of an individual good) if the following condition is met:

Condition 7. $\frac{l}{m}v_L + \frac{m-l}{m}v_H - c_H > v_L - c_L$

Given the starting assumptions, if Condition 7 is satisfied then the following proposition holds:

Proposition 10. *If Condition 7 is satisfied (in addition to the starting assumptions), then the equilibrium with the greatest quantity sold in the lemons market will have greater consumer surplus than the perfect information equilibrium.*

Proof. As shown above, given the starting assumptions, identical buyers will receive consumer surplus equal to $v_L - c_L$ under perfect information. Under the lemons market, the equilibrium with the greatest quantity sold will be one such that m low quality goods and $m - l$ high quality goods trade. Notice that Condition 7 implies Condition 6: from the starting assumptions $v_L - c_L \geq 0$, which means if $\frac{l}{m}v_L + \frac{m-l}{m}v_H - c_H > v_L - c_L$, then $\frac{l}{m}v_L + \frac{m-l}{m}v_H - c_H \geq 0$. This requires that Condition 6 be satisfied. At this equilibrium with the greatest quantity traded, expected buyer value is $\frac{l}{m}v_L + \frac{m-l}{m}v_H$ and market price is c_H at the equilibrium with the greatest quantity sold, with consumer surplus equal to the difference of the two. If that difference $\frac{l}{m}v_L + \frac{m-l}{m}v_H - c_H$ is greater than $v_L - c_L$, then the consumer surplus in the lemons market is greater than the consumer surplus of the perfect information market. $\square$

Proposition 10 defines when the equilibrium with the highest quantity sold in the lemons model results in greater consumer surplus than the perfect information case. However, if consumers are never indifferent to purchasing with only low quality goods in the market, then the equilibrium will be unique, as seen in the section above. This unique equilibrium has higher consumer surplus than the perfect information equilibrium if Condition 7 holds.

This leads to Corollary 2:

Assumption 3. *$v_L > c_H$, the value to the buyers of the low quality good is greater than the production cost of the high quality product.*

Corollary 2. *If Assumption 3 is satisfied in addition to the starting assumptions and Condition 7, then there will be unique competitive equilibrium in the lemons market, which has greater consumer surplus than the perfect information equilibrium.*

Proof. The proof of Corollary 2 comes from the discussion of Condition 6 at the start of the section. As shown there, if $v_L > c_H$, there is only one equilibrium in the lemons market. Therefore, if Assumption 3 is satisfied, there is a unique equilibrium in the lemons market and with Condition 7, that equilibrium produces greater consumer surplus than the perfect information case. □

A.7 gives an example of how consumer surplus can decrease when moving from the lemons market to perfect information with two quality types.

6.1.1 More Than Two Types

So far, this section has consider the case of two quality types, denoted high and low, and how consumer surplus can be greater under the lemons model than under perfect information. Now I generalize the result for more than two types. The assumptions above are maintained, however it is also assumed that gains to trade are non-decreasing in quality.[16] Let the quality types be a discrete number of quality levels in the range $\omega \in [\underline{\omega}, \overline{\omega}]$.

Let ω^* be the lowest quality type traded under perfect information. Since it is assumed the gains to trade are increasing in ω, in the perfect information equilibrium the highest types beginning with type $\overline{\omega}$ will be traded until m total goods are traded. The mth good traded is then of type ω^*. With m identical consumers, all consumers receive the same consumer surplus, which is based on the lowest traded quality, ω^*. Since ω' is the marginal type, sellers of ω^* must be indifferent between trading and not trading, so the price of an ω^* quality good is equal to the seller's cost, $p_{\omega^*} = c_{\omega^*}$. The consumer surplus for an individual consumer then must be $v_{\omega^*} - c_{\omega^*}$, and aggregate consumer surplus is $m * (v_{\omega^*} - c_{\omega^*})$.

Under the lemons model, all the lowest types trade, as long as value to the consumers of the m traded low quality goods is higher than the cost of the highest traded quality. Let ω' signify the highest quality traded good: all goods with quality less than ω' trade, some goods of quality ω' trade, and no goods with quality greater than ω' trade. This establishes

[16]This was previously assumed with just the two quality types; now it is assumed to hold for all possible number of types.

the market price: the price will be such that a seller of type ω' is indifferent, $p = c_{\omega'}$. Let n_ω be the measure of goods of quality type ω. Given the price, consumer surplus can be expressed as the difference between the value of each type traded minus the common price, $c_{\omega'}$:

$$\sum_{\underline{\omega}}^{\omega'-1} n_\omega(v_\omega - c_{\omega'}) + (i - \sum_{\underline{\omega}}^{\omega'-1} n_\omega)(v_{\omega'} - c_{\omega'})$$

The consumer surplus is based on the difference between the value of each type traded and the cost of the marginal type. For consumer surplus to be greater with less information, it is required that:

$$\sum_{\underline{\omega}}^{\omega'-1} n_\omega(v_\omega - c_{\omega'}) + (i - \sum_{\underline{\omega}}^{\omega'-1} n_\omega)(v_{\omega'} - c_{\omega'}) > i * (v_{\omega^*} - c_{\omega^*})$$

When will this condition be satisfied? The two greatest drivers will be the difference between v_{ω^*} and c_{ω^*}, and the difference between each v_ω and $c_{\omega'}$. The first difference defines the consumer surplus of the perfect information case: the smaller the gains to trade of the lowest quality type traded, the smaller the total consumer surplus of the perfect information case. The second difference defines the consumer surplus of standard lemons case: if the cost of the marginally traded type is low relative to the average value of the traded types, consumer surplus will be high in the lemons case.

One final condition for consumers to receive greater surplus under the lemons case is that the highest quality traded type of the lemons case, ω', must be higher than the lowest quality traded type of the perfect information case, ω^*. In other words, there must be an intersection of types sold between the lemons case and the perfect information case. If $\omega^* > \omega'$, the perfect information case provides greater consumer surplus. This is because of the assumption of increasing gains to trade as quality increases. If $\omega^* > \omega'$, the consumer surplus for the best quality type in the lemons case is less than the consumer surplus for any quality in the perfect information case. Since any quality type less than ω' produces less consumer surplus than ω', each trade under the standard lemons case produces less consumer surplus than each trade under perfect information. Therefore, $\omega^* < \omega'$ is a necessary condition for consumers to receive more surplus in the standard lemons case.

6.2 Common Signal

6.2.1 General Case

Property 1. *If fewer trades occur after the signal, consumer surplus must be non-increasing.*

Property 1 could be made more general: consumer surplus must be zero when fewer than m trades occur. This is a result of the buyers' identical values: all buyers receive the same consumer surplus, so if some buyers are not purchasing, all buyers receive zero consumer surplus.

6.2.2 Two Quality Case

For the two quality case, consumer surplus will decrease when total welfare decreases. This gives a corollary to Proposition 5:

Corollary 3. *If Assumptions 1 and 2 are satisfied, then the equilibrium with highest quantity sold in the standard lemons market yields higher consumer surplus than when consumers receive an informative signal.*

Proof. In the standard lemons market, in the equilibrium with highest quantity sold buyers pay a price of c_H and receive a good with greater expected value than that price. Consumers will have positive expected surplus. Under Assumptions 1 and 2, the equilibrium under the common signal gives consumer surplus equal to zero. The reason for this is that fewer than m trades occur, as discussed in the proof of Proposition 2. If fewer than m trades occur, then since all consumers are identical and some consumers are unable to trade, all must receive consumer surplus of zero.

□

6.3 Individual Buyer Information

With a fraction of informed buyers, in the two quality case the necessary conditions for consumer surplus to decrease as a result of the informed buyers are similar to those for total welfare to fall: $m - r > l$ and Condition 5. The following condition on the parameters must also hold:

Condition 8. $l * v_L + (m - l) * v_H - c_H > r * (v_H - c_H)$

The left side of the inequality is the total consumer surplus under the lemons market given m goods trade: the price paid is c_H and the value is the sum of l low quality goods and $m - l$ high quality goods. The right side of the inequality is the total consumer surplus with a fraction of perfectly informed buyers, given $m - r > l$: the $m - r$ uninformed buyers receive no consumer surplus and the r informed buyers each receive consumer surplus equal to the value of the high quality minus the cost.

Corollary 4. *If $m - r > l$ and Conditions 5 and 8 are satisfied, then consumer surplus is lower with a fraction of informed consumers than under the standard lemons market.*

Proof. If expected value with l low quality goods and $m - r - l$ high quality goods is less than c_H (Condition 5), then uninformed consumers will be unwilling to purchase at price c_H, the price that brings high quality sellers into the market. In this case, l uninformed buyers will buy low quality cars at a price of v_L and $m-r-l$ uninformed consumers will be unable to purchase. As a result, all uninformed buyers receive consumer surplus equal to zero. The best that informed buyers can do is pay a price of c_H, the lowest price high quality sellers will accept, and receive a good valued at v_H. Total consumer surplus is then $r*(v_H - c_H)$, which is less than under the lemons market, under Condition 8. □

7 Conclusion

Solutions to adverse selection issues frequently rely on increasing the information available to the uninformed parties. Here, I have shown that increasing the information to buyers in a lemons market does not necessarily increase total welfare, nor does it necessarily increase consumer surplus. Not only are informational increases insufficient to ensure total welfare increases, but also increasing the volume of trade is not able to ensure an increase in total welfare. Information that increases the volume of trade may also shift the composition of traded goods away from goods with higher gains to trade to goods with lower gains to trade.

These results point to an important conclusion: neither increases in information nor increases in volume of trade guarantee that welfare is also increasing, and thus are not sufficient measures of increased total welfare in adverse selection markets. Policy makers in such contexts must be aware that (unlike in Levin [2001]) the value of a policy cannot be determined simply by observing the levels of trade before and after an informational change. Also, as is consistent with Levin (2001), neither is an informational increase sufficient to improve total welfare, as particularly demonstrated in the two quality case where informational gains are unambiguous.

These results suggest that policies that increase information to the consumer have the potential to be harmful, both to total efficiency and even to consumers themselves. Even if these policies lead to an increases in the number of transactions, there is still uncertainty about whether the policies are ultimately welfare-improving.

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A Examples

A.1 Signal Refinement of Single Subgroup Causes Increased Trade and Decreased TW

This examples shows how the refinement of a single subgroup can cause the volume of trade to increase, but total welfare to decrease. A necessary condition was that measure of buyers be less than measure of sellers: in this example, 8 buyers and 10 sellers. The following table gives the full distribution of goods in the market, three quality types with seller costs and buyer valuations:

Type	Value	Cost	# of sellers
1	2	1	5
2	6	4	3
3	10	6.5	2

Suppose the original partition is {4,3,0} {1,0,2}. What goods sell under the original partition?

The first subgroup has an expected buyer value of $3.7 = \frac{(4*2+3*6)}{7}$, which means there is no price that will encourage type #2 sellers to trade and also encourage buyers to buy. Thus only the four type #1 sellers in the first partition sell.

In the second subgroup, the expected buyer value of all goods is $7.33 = \frac{(1*2+2*10)}{3}$, which means that all the goods can be sold. What are the total gains to trade with two subgroups? Five type #1 goods traded with gains of one, no type #2 goods are traded, and two type #3 with gains of three and a half means a total of 12 units of total welfare gained from 7 trades.

Suppose that the first subgroup is further partitioned: {(4,0,0)(0,3,0)}{(1,0,2}. The new partitioning of the first subgroup allow both types to be traded, for the first sub-subgroup at a price between 1 and 2, and for the second at a price between 4 and 6 (greater than seller's cost and less than buyer's valuation). This means that sellers could offer the buyers in the first sub-subgroup up to one unit of consumer surplus and in the second, up to two units of consumer surplus. In the other subgroup, average buyer value was 7.33 and the lowest price that would keep the highest quality sellers in the market is 6.5 (seller cost). This leaves a maximum possible consumer surplus of .833, less than what buyers can obtain from either of the newly partitioned sub-subgroups. Since there are only 8 buyers, this means there is not a sufficient measure of consumers to continue buying the highest quality in the second subgroup. Instead the eight buyers purchase five type #1 goods and three type #2 goods, with consumer surplus per buyer between .83 (the amount that would allow the highest quality sellers back in the market) and 1 (the limit of consumer surplus for type #1 buyers).

The volume of trade has clearly increased, from seven to eight. However, what has happened to total welfare? Five type #1 goods traded with gains of one, three type #2 goods are traded with gains of two, and no type #3 are traded, meaning a total of 11 units of total welfare gained from the 8 trades.

The additional partition of the subgroup has led to an increase in the overall volume of trade, however because lower quality goods are taking the place of high quality goods, total welfare in the entire market decreases as a result of the additional information.

A.2 Signal Refinement of Multiple Subgroups Causes Increased Trade and Decreased TW

In this example, multiple subgroups are further refined, leading to an increase in the volume of trade, but a decrease in total welfare. Unlike the previous case, this does not require that the measure of buyers be less than the number of sellers. Here there are 13 sellers, and number of buyers can be any amount greater than or equal to 12. The following table gives the full distribution of goods in the market, three quality types with seller costs and buyer valuations:

Type	Value	Cost	# of sellers
1	1	0	7
2	4	2	4
3	10	4.5	2

Suppose the original partition is {5,2,0}{2,2,2}. What goods sell under the original partition?

The first subgroup has an expected buyer value of $1.86 = \frac{(5*1+2*4)}{7}$, which means there is no price that will encourage type #2 sellers to trade and also encourage buyers to buy. Thus only the five type #1 sellers in the first partition sell.

In the second subgroup, the expected buyer value of all goods is $5 = \frac{(2*1+2*4+2*10)}{6}$, which means that all the goods can be sold. What are the total gains to trade with two subgroups? Seven type #1 goods traded with gains of one, two type #2 goods traded with gains of two, and two type #3 with gains of five and a half means a total of 22 units of total welfare gained from 11 trades.

Suppose that both subgroups are further partitioned: {5,0,0)(0,2,0)}{(2,2,1)(0,0,1)}. The new partitioning of the first subgroup allow both types to be traded, for the first sub-subgroup at a price between 0 and 1, and for the second at a price between 2 and 4 (greater than seller's cost and less than buyer's valuation). The first sub-subgroup of the second group now has an average buyer value of $4 = \frac{(2*1+2*4+1*10)}{5}$, which does not allow for this subgroup to trade at a price that includes type #3 sellers. As a result, the first sub-subgroup trades only the two type #1 and the two type #2 goods, and the second trades the only good in it, the type #3 good. Across all groups, seven type #1 goods, four type #2 goods, and one type #3 good are sold, leading to a total of 20.5 units of total welfare gained from 12 trades.

The additional partitions of the subgroups have led to an increase in the overall volume of trade, however because lower quality goods are taking the place of high quality goods, total welfare in the entire market decreases as a result of the additional information.

A.3 Increased Information Decreasing TW From Lemons Market

The following example demonstrates the lack of monotonicity in total welfare and consumer surplus when information about the quality of goods increase. In this example, the signal will be knowing for a select number of high quality goods that they are high quality. The belief about quality for the remaining goods is just the adjusted proportion after the certain high quality goods are removed from the group.

Let $m = 3$, $n = 4$, and $i = 6$. Let n_b be the number of high quality goods that are lumped into group B, the group with lower expected quality than the initial proportion, which can vary from zero to four. At zero, there is perfect information: consumers know exactly which good is in which group. At four, there is the standard lemons model: consumers only know the overall distribution of quality. As n_b decreases, the information the consumers has increases: they know for certain a higher number of high quality goods, and they have a better prediction which goods are low quality.

Let the quality of low type be equal to 10, and the quality of the high type be equal to 30. Consumers have values equal to $2 * \omega$, and sellers have costs equal to $1.3 * \omega$.[17]

The graph below gives the total welfare and consumer surplus that results from each realization of n_b. The graph starts on the left where $n_b = 0$, that is where no high quality goods are lumped into a group with low quality goods: perfect information. As mentioned previously, perfect information will always maximize total welfare, as here total welfare peaks at 96 (four high quality goods traded with 21 units of gains to trade per transaction, and two low quality goods traded with 7 units of gains to trade per transaction). Of those 96 units of total welfare, 42 are captured by the consumers in consumer surplus.

On the far right side is the other extreme: the standard lemons case, where all goods are lumped into one group from the perspective of the buyers, and the only information is the underlying proportion of high and low quality. Here, total welfare is 84 (three high quality goods traded with 21 units of gains to trade per transaction, and three low quality goods traded with 7 units of gains to trade per transaction). Of those 84 units, only 5 units are captured by buyers as consumer surplus.[18]

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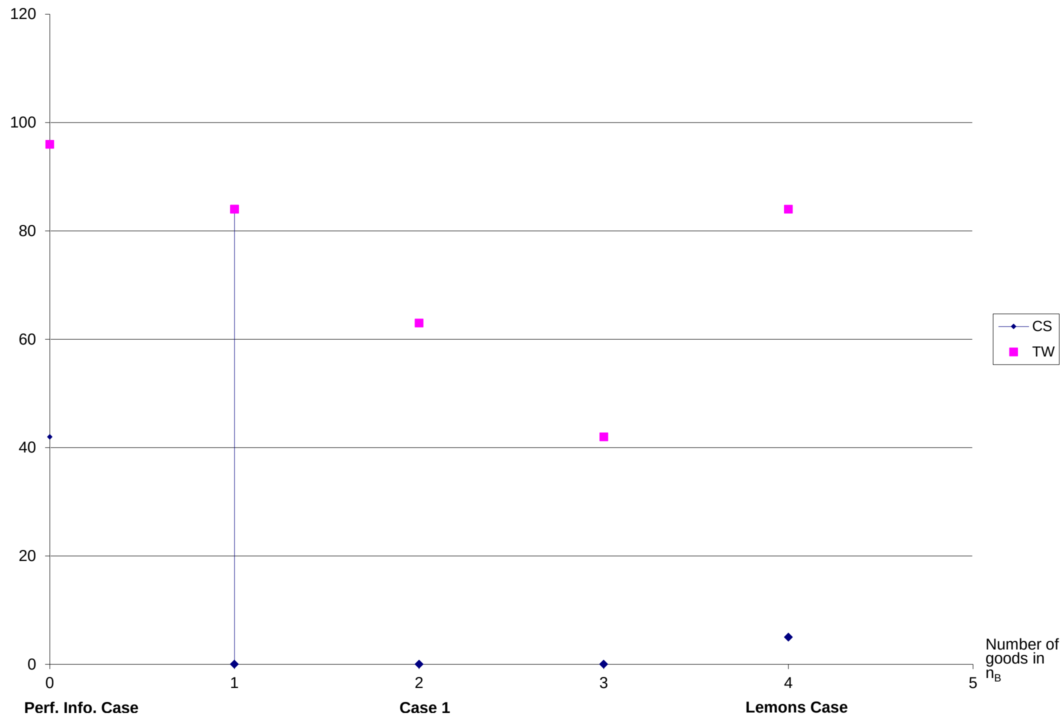
[17] This makes $v_H = 60$, $v_L = 20$, $c_H = 39$, and $c_L = 13$.

[18] Note that this means Proposition 6 cannot be satisfied. Condition 2 does not hold: $\frac{m}{i}v_L + \frac{m-l}{i}v_H - c_H = 1$ and $v_L - c_L = 7$. ($\frac{3}{6}20 + \frac{3}{6}60 - 39 = 1$ and $20 - 13 = 7$.)


120
100
80
60
40
20
0
0
1
2
3
4
5
CS
TW
Number of goods in n_B
Perf. Info. Case
Case 1
Lemons Case

The information possessed by consumers increases with movement from the right side of the graph to the left side of the graph. As the graph demonstrates, neither welfare nor consumer surplus are monotonically increasing or decreasing as information increases. When information is maximized in the perfect information case, total welfare is necessarily maximized, but as information increases from the lemons case to cases less than perfect information, total welfare (and consumer surplus) can fall as a result.

A.4 Increased Mean Difference and Trade Decreasing TW, #1

This example shows how increasing the mean values of the subgroups leading to an increase in trade can nevertheless lead to a decrease in total welfare. There are five quality types, delineated by their type, 1-5, where a higher type indicates higher value to buyer and greater cost to the seller. Each quality type has a different value to the buyer and cost to the seller, and gains to trade are non-decreasing in quality. The following table gives the quality types with their values and costs, and the total number of each type:

Type	Value	Cost	# of sellers
1	1	0	13
2	4	2	7
3	11	4	6
4	14	6	1
5	18	8	4

Assume there are at least 25 potential buyers. The signal results in the following distribution:

Group A			Group B	
Type	# of sellers		Type	# of sellers
1	8		1	5
2	3		2	4
3	0		3	6
4	1		4	0
5	4		5	0
Avg. Value	**6.6**			**5.8**

What goods will be sold under this signal? For group A, notice that average value with all A goods in the market is 6.6, less than 8, the cost of the highest, quality #5 sellers. Thus, the highest quality will not sell in group A. Eliminating those four sellers gives a new average value of 2.8, lower than the cost of the second highest, quality #4 seller. When she too drops out of the market, the average value in the market drops to 1.8. At this value, there is no acceptable price for buyers than will keep the quality #2 sellers in the market. As a result, the only goods traded from group A are the lowest quality types, #1, all of which will trade.[19]

For group B, the average value is above the cost of the highest type in the group, type #3, so all goods in group B will sell. The change in total welfare after trade is simply the total of the differences between buyer's valuations and seller's costs. 13 type #1 trades each produce 1 unit of welfare, 4 of type #2 produce 2 units each, and 6 of type #3 produce 7 units each. Total welfare gains are then $13*1+4*2+6*7 = 63$ units, with 23 trades occurring.

Suppose the signal changes, moving four of the type #3 from group B to group A. The distribution is:

Group A			Group B	
Type	# of sellers		Type	# of sellers
1	8		1	5
2	3		2	4
3	4		3	2
4	1		4	0
5	4		5	0
Avg. Value	**7.5**			**3.9**

Notice first that difference in the mean values has increased: the average quality in group A has increased from 6.6 to 7.5 and the average quality in group B has decreased from 5.8 to 3.9. What trades occur now?

In group A, the average value of all goods is still below the cost of the highest type, thus all #5 sellers drop out of the market. Once the highest quality sellers drop out of the market, the average value of the remaining goods drops to 4.9. This is insufficient to keep the #4 seller in the market, and she too leaves. With only the three lowest types in the market for group A, average value is 4.3, which means that all remaining goods can be sold, at a price between 4 (the cost of the type #3) and 4.3 (buyer's average value).

In group B, the loss of average value means that there is no longer a price that consumers would accept greater than the cost of the type #3 seller (cost is still 4, average value is only 3.9). With the type #3 sellers out of the market, the average value drops to 2.3, however

[19] Since there are assumed to be at least 25 buyers, and as will be shown further in this example, fewer than 25 trades occur all buyers receive CS = 0. This means that the price of goods in group A is the value of the lowest type, $p_A = 1$. Since the price is only a transfer from buyers to sellers, it is irrelevant in the total welfare analysis.

this value is sufficient to keep all other types in market, trading a price between 2 (the cost of the type #2) and 2.3 (buyer's average value).

What is the change in total welfare under the new signal? Group A trades 8 type #1, 3 type #2, and 4 type #3, and group B trades 5 type #1 and 4 type #2. Total welfare increase is then $13 * 1 + 7 * 2 + 4 * 7 = 55$ units, with 24 trades occurring.

The difference of the mean values has increased from the first signal to the second: the average value in A increased and in B decreased. This lead to an increase in the total volume of trade: 23 trades in the old signal compared to 24 trades in the new signal. However, despite these increases in trade and difference in means, total welfare from trade fell, from 63 units to 55 units.

Note that this can only occur because of the difference in average value of the traded types under the old signal: group A could only trade the lowest type, but group B trade the three lowest types. The resulting loss in total welfare comes from how increasing the average value of group A led to only selling more type #2, at the cost of making it impossible for group B to trade type #3 as it had before. This results in more trade, but it exchanges trades with high gains for trades with lower gains. Thus, even though the volume of trade increases, total welfare falls as the decrease in the quality of traded goods outweighs the increase in quantity of trade.

A.5 Increased Mean Difference and Trade Decreasing TW, #2

This example shows a parallel case to the previous example: the difference in the means increases, causing trade to increase but total welfare to decrease. However, in this case, the additional low quality trades after the signal change occur in the low signal group, B, instead of in the high signal group, A. Even so, the same necessary condition holds: the average value traded of the group with lower traded value before the signal change must increase and the average value traded of the group with higher traded value before the signal change must decrease.

There are four quality types, delineated by their type, 1-4, where a higher type indicates higher value to buyer and greater cost to the seller. Each quality type has a different value to the buyer and cost to the seller, and gains to trade are non-decreasing in quality. The following table gives the quality types with their values and costs, and the total number of each type:

Type	Value	Cost	# of sellers
1	1	0	13
2	4	2	6
3	11	4	5
4	18	8	2

Assume there are at least 23 potential buyers. The signal results in the following distribution:

Group A			Group B	
Type	# of sellers		Type	# of sellers
1	5		1	8
2	3		2	3
3	5		3	0
4	0		4	2
Avg. Value	**5.5**			**4.3**

What goods will be sold under this signal? For group A, notice that average value with all A goods in the market is 5.5, greater than, the cost of the quality #3 sellers, highest in the group. Thus, all three existing quality types in group A will sell, at a price between 4 (cost of #3) and 5.5 (buyer's average valuation).

For group B, the average value is below the cost of the highest type in the group, type #4, so the two sellers of type #4 will drop out of the market. Once those two sellers leave the market, the new average value is 1.8, which is insufficient to cover the costs of the type #2 seller. This means that the only goods sold in group B are the lowest quality type, #1. Total welfare gains are then $13*1+3*2+5*7 = 54$ units, with 21 trades occurring.

Suppose the signal changes, moving two of both type #2 and #3 from group A to group B, and moving both the type #4 goods from group B to group A. The distribution now is:

Group A			Group B	
Type	# of sellers		Type	# of sellers
1	5		1	8
2	1		2	5
3	3		3	2
4	2		4	0
Avg. Value	**7.1**			**3.3**

Notice first that difference of the mean values has increased: the average quality in group A has increased from 5.5 to 7.1 and the average quality in group B has decreased from 4.3 to 3.3. What trades occur now?

In group A, the average value of all goods is still below the cost of the highest type, thus all #4 sellers drop out of the market. Once the highest quality sellers drop out of the market, the average value of the remaining goods drops to 4.7. This is sufficient to keep the #3 sellers in the market, which means that all remaining goods can be sold, at a price between 4 (the cost of the type #3) and 4.7 (buyer's average value).

In group B, the average value is not enough to allow a price that would keep type #3 sellers in the market. With the type #3 sellers out of the market, the average value drops to 2.2, however this value is sufficient to keep all other types in market, trading a price between 2 (the cost of the type #2) and 2.2 (buyer's average value).

What is the change in total welfare under the new signal? Group A trades 5 type #1, 1 type #2, and 3 type #3, and group B trades 8 type #1 and 5 type #2. Total welfare increase is then $13 * 1 + 6 * 2 + 3 * 7 = 46$ units, with 22 trades occurring.

The difference of the mean values has increased from the first signal to the second: the average value in A increased and in B decreased. This lead to an increase in the total volume of trade: 21 trades in the old signal compared to 22 trades in the new signal. However, despite these increases in trade and the difference of mean values, total welfare from trade fell, from 54 units to 46 units.

Note that this can only occur because of the difference in average values of the traded types under the old signal: group B could only trade the lowest type, but group A trade the three lowest types. The resulting loss in total welfare comes from how increasing the average traded value of group B led to only selling more type #2, while group A has fewer type #3 goods to trade. This results in more trade, but it exchanges trades with high gains for trades with lower gains. Thus, even though the volume of trade increases, total welfare falls as the decrease in the quality of traded goods outweighs the increase in quantity of trade.

A.6 Increased Trade and Decreased Mean Difference Decreasing TW

The previous examples demonstrated how increasing the difference of the mean values of the subgroups and the volume of trade could actually hurt total welfare, but only if the average value traded of the group with lower traded value before the signal change increases and the average value traded of the group with higher traded value before the signal change decreases. This example demonstrates that that can occur even more easily with a decrease in the difference of the mean values of the subgroups; it is easier to meet the necessary condition of increasing the low and decreasing the high average traded values. The example is similar to the previous ones: there are three quality types, delineated by their type, 1-3, where a higher type indicates higher value to buyer and greater cost to the seller. Each quality type has a different value to the buyer and cost to the seller, and gains to trade are non-decreasing in quality. The following table gives the quality types with their values and costs, and the total number of each type:

Type	Value	Cost	# of sellers
1	1	0	6
2	4	2	3
3	8	3	2

Assume there are at least 11 potential buyers. The signal results in the following distribution:

Group A			Group B	
Type	# of sellers		Type	# of sellers
1	1		1	5
2	1		2	2
3	2		3	0
Avg. Value	**5.25**			**1.85**

What goods will be sold under this signal? For group A, notice that average value with all A goods in the market is 5.3, greater than 3, the cost of the highest, quality #3 sellers. Thus, it is possible for all sellers to sell at a price between 3 (cost of #3 sellers) and 5.3 (buyer's average value).

For group B, the average value is not above the cost of the highest type in the group, type #2, so all sellers of #2 will drop out of the market. In B, only the lowest quality types will sell.

The total goods sold across A and B will be 6 of type #1, 1 of type #2, and 2 of type #3. With gains to trade of 1, 2, and 5 for types #1, #2, and #3 respectively, this gives a total welfare increase due to trade of $6 * 1 + 1 * 2 + 2 * 5 = 18$ units, with 10 trades.

Suppose the signal changes, moving one of each type #2 and #3 to the group B. The distribution is:

Group A			Group B	
Type	# of sellers		Type	# of sellers
1	1		1	5
2	0		2	3
3	1		3	1
Avg. Value	**4.5**			**2.8**

What goods will be sold under this signal? For group A, notice that average value with all A goods in the market is 4.5, greater than 3, the cost of the highest, quality #3 sellers. Thus, it is possible for both sellers to sell at a price between 3 (cost of #3 sellers) and 4.5 (buyer's average value).

For group B, the average value is not above the cost of the highest type in the group, type #3, so the seller of #2 will drop out of the market. The average value after the #3 seller drops out of the market is 2.1. All #1 and #2 sellers can sell at a price between 2 (cost of type #2) and 2.1 (buyer's average value).

The total goods sold across A and B will be 6 of type #1, 3 of type #2, and 1 of type #3. With gains to trade of 1, 2, and 5 for types #1, #2, and #3 respectively, this gives a total welfare increase due to trade of $6 * 1 + 3 * 2 + 1 * 5 = 17$ units, with 11 trades.

As in the previous example, the volume of trade increased while the total welfare produced by trade decreased. In this case, it did not depend on the low average value group (B) selling higher quality types than A under the first signal. Instead, the shift of several higher quality goods from group A to B allowed for an increase in the number of trades of the medium quality type (#2), but at the cost of a trade of the highest quality type (#3). Since the gains to trade are so much higher for type #3 than for type #2, the loss of welfare from the shift in composition of trade outweighed the gain of welfare from increased volume of trade.

A.7 CS in Lemons Market

This example demonstrates how the lack of information of the lemons market can lead to greater consumer surplus than the perfect information case.

There are four buyers and five sellers, two sellers of low quality and three sellers of high quality. This satisfies three of the needed assumptions: $i > m$, $i > n$, and $i < n + m$. The low quality has quality equal to 1, and the high quality has quality equal to 3. Buyers have quality for each type equal to $4 * quality$, and sellers have a cost of for each type equal to $1 * quality$.[20] This satisfies the rest of our assumptions: $v_L > c_L$, $v_H > c_H$, and

[20] This makes $v_L = 4$, $v_H = 12$, $c_L = 1$, $c_H = 3$.

$v_L > c_H$. Finally, Condition 6 is satisfied: $\frac{2}{4} * 4 + \frac{2}{4} * 12 - 3 > 4 - 1 => 8 - 3 > 4 - 1 => 5 > 3$.

Condition 6 is a comparison of the consumer surplus for perfect information and the standard lemons model. In the lemons model equilibrium, four goods are traded, two high quality and two low quality, at a price equal to the cost of producing the high quality, $p = 3$. The total value is the value of the four goods traded (4+4+12+12), 32. The average value per traded good is then 8, which at a price of 3 means each consumer receives on average 5 units of consumer surplus, for 20 total units of consumer surplus in the lemons model. In perfect competition, four goods are also traded, this time however three high quality goods and one low quality good. The low quality good is traded at price of 1, and the high quality goods are traded at a price of 9. Consumer surplus per buyer is 3, for 12 units of total consumer surplus. Thus, as expected from Proposition 10, consumer surplus is greater in the lemons model than under perfect information ($20 > 12$).

Finally, even though consumer surplus was higher under the lemons case, total welfare is still higher under perfect information. Under perfect information, the gains to trade are $3 * (12 - 3) + 1 * (4 - 1) = 30$, three high quality goods traded with gains to trade of 9 and one high quality good with gains to trade of 3. Under the lemons model, the gains to trade are $2 * (12 - 3) + 2 * (4 - 1) = 24$, two high quality goods traded with gains to trade of 9 and two low quality goods traded with gains to trade of 3.